Asymmetric distributions for random density matrices

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Problem Statement

Originating in Wishart's work on sample covariance matrices [Wis28], the theory of random matrices [AGZ10, MS17] is a well developed subfield of mathematics, having interactions with many many other research areas, such as theoretical physics, computer science and telecommunication, statistics, etc. In the recent years, the theory of quantum information [Wat18] has also benefited from random matrix models, both as a way to study the properties of typical objects (like states and channels) and also as a source of extremal examples [CN16]. The most studied such model is that of *random density matrices* [SŻ04]: one is interested in random matrices ρ which are positive semidefinite $\rho \geq 0$ and have unit trace $\operatorname{Tr} \rho = 1$. All the models in the literature are symmetric, in the sense that the distribution of the random matrix is invariant under permutations [ŻPNC11]. However, in quantum information theory one is met in most situations with specific conditions which do not posses this kind of symmetry, and some basis elements (corresponding to classical states) should have a priori larger weights.

Goal of the internship

The candidate will start with the study of *Dirichlet distributions*. These distributions play an important role in statistics, mainly as conjugate priors of multinomial distributions. The candidate will take the point of view of information theory [Mac03], and study them as probability distributions on the set of *classical states* (that is, probability vectors). The candidate will then generalize Dirichlet distributions to *quantum states*, in connection to the Wishart ensemble [Nec07] and the *compound Wishart ensemble* [MS17]. To do this, the candidate will first get acquainted with quantum theory, then develop a model for *asymmetric random quantum states*, and finally study their statistical properties in relation to the classical Dirichlet distributions.

Candidate's profile

The candidate should have a strong mathematical profile, with a focus on statistics and probability theory. Competences in random matrix theory, linear and multilinear algebra, quantum (information) theory, scientific software (`python` or `MATLAB`) are the most important for the research project (but not strictly required, the missing ones will be acquired during the internship).

There is the possibility to continue on to a **PhD thesis** on similar topics at the end of the Master's project.

References

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[Nec07] Ion Nechita. Asymptotics of random density matrices. *Annales Henri Poincaré*, 8(8):1521–1538, 2007.

[SŻ04] Hans-Jürgen Sommers and Karol Życzkowski. Statistical properties of random density matrices. *Journal of Physics A: Mathematical and General*, 37(35):8457, 2004.

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