# Spectrum and asymptotical freeness for block-modified random matrices

Supervisor: Ion NECHITA
Department: LPT Toulouse
Email: nechita@irsamc.ups-tlse.fr
Webpage: http://ion.nechita.net/about

## Problem Statement

By now, the theory of random matrices [AGZ10] is a well developed subfield of mathematics, having interactions with many many other research areas, such as theoretical physics, computer science and telecommunication, operator algebras, combinatorics, number theory, etc. In the recent years, the theory of quantum information [Wat18] has also benefited from random matrix models, both as a way to study the properties of typical objects (like states and channels) and also as a source of extremal examples [CN16]. In quantum theory, the tensor product of Hilbert spaces is a central operation, allowing to take into considerations systems composed of several components. Hence, the study of random operators acting on a tensor product $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2}$ is central to the theory. In particular, the study of the *partial action* of a linear map on such bi-partite operators is very important in the study of *quantum entanglement* [HHHH09], one of the most important features of quantum theory. Such matrix models, called *block-modified random matrices* have been studied in [ANV16], where the asymptotic spectral distribution has been computed in some particular situations. The main technique used there was the notion of operator-valued freeness developed by Voiculescu and Speicher [Spe98].

## Goal of the internship

The candidate will continue the development of the theory of block-modified random matrices in two directions. First, identify the most general cases where the asymptotical spectrum of the modified matrix can be computed from the initial data (distribution of the random matrix and the linear map acting on a tensor factor). Afterwards, study the *joint distribution* of two different modifications of a random matrix and eventually prove freeness results. The theoretical results will be applied to the study of entanglement in quantum information.

## Candidate's profile

The candidate should have a strong mathematical profile, with a focus on probability theory and random matrix theory. Competences in linear and multilinear algebra, operator algebras, quantum (information) theory are the most important for the research project (but not strictly required). Some numerical aspects of the random matrix models will be discussed, so familiarity with scientific software (`python` or `MATLAB`) could be useful.

There is the possibility to continue on to a **PhD thesis** on similar topics at the end of the Master's project.

## References

[AGZ10] Greg W Anderson, Alice Guionnet, and Ofer Zeitouni. *An introduction to random matrices*. Cambridge University Press, 2010.

[ANV16] Octavio Arizmendi, Ion Nechita, and Carlos Vargas. On the asymptotic distribution of block-modified random matrices. *Journal of Mathematical Physics*, 57(1):015216, 2016.

[CN16] Benoit Collins and Ion Nechita. Random matrix techniques in quantum information theory. *Journal of Mathematical Physics*, 57(1), 2016.

[HHHH09] Ryszard Horodecki, Paweł Horodecki, Michał Horodecki, and Karol Horodecki. Quantum entanglement. *Reviews of Modern Physics*, 81(2):865, 2009.

[MS17] James A Mingo and Roland Speicher. *Free probability and random matrices*, volume 35. Springer, 2017.

[Spe98] Roland Speicher. *Combinatorial theory of the free product with amalgamation and operator-valued free probability theory*, volume 627. American Mathematical Soc., 1998.

[Wat18] John Watrous. *The Theory of Quantum Information*. Cambridge University Press, 2018.