Incompatibility of quantum measurements

Supervisor: Ion NECHITA
Department: LPT Toulouse
Email: nechita@irsamc.ups-tlse.fr
Webpage: http://ion.nechita.net/about

Problem Statement

One of the defining properties of quantum mechanics is the existence of incompatible observables, i.e. measurements that cannot be performed simultaneously, with the observables of position and momentum as a classic example. One of the central notions to capture this property of quantum mechanics is *joint measurability* or *compatibility*. Quantum measurements are compatible if they arise as marginals from a common measurement. Compatibility has practical implications for quantum information tasks [BCP+14], as only incompatible measurements can violate Bell inequalities. Therefore, incompatibility can be seen as a resource for quantum information tasks similar to entanglement [HMZ16]. It is well-known that incompatible measurements become compatible if enough noise is added to them. While many works study compatibility questions for concrete measurements, there has also been an interest in how much incompatibility there is in quantum mechanics and other generalized probabilistic theories [BHSS13].

Goal of the internship

The candidate will start with the study of the different examples of maximally incompatible sets of measurements in the literature. In the light of recent results in the field [BN18], many cases remain open, both in the binary situation (quantum effects) and in the general case. Using both numerical [BQG+17] and analytical methods, the candidate will investigate new cases of *maximally incompatible quantum observables*. The notions of *mutually unbiased bases* [WF89] and *random quantum measurements* [CN16] will certainly play an important role in this task.

Candidate's profile

The candidate should have a good grasp of quantum mechanics and linear algebra. Competences in quantum information theory, and scientific software (`python` or `MATLAB`) are useful (but not required, the missing ones will be acquired during the internship).

There is the possibility to continue on to a **PhD thesis** on similar topics at the end of the Master's project.

References

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[BN18] Andreas Bluhm and Ion Nechita. Joint measurability of quantum effects and the matrix diamond. *Journal of Mathematical Physics*, 59(11):112202, 2018.

[BQG+17] Jessica Bavaresco, Marco Túlio Quintino, Leonardo Guerini, Thiago O Maciel, Daniel Cavalcanti, and Marcelo Terra Cunha. Most incompatible measurements for robust steering tests. *Physical Review A*, 96(2):022110, 2017.

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[HMZ16] Teiko Heinosaari, Takayuki Miyadera, and Mário Ziman. An invitation to quantum incompatibility. *Journal of Physics A: Mathematical and Theoretical*, 49(12):123001, 2016.

[WF89] William K Wootters and Brian D Fields. Optimal state-determination by mutually unbiased measurements. *Annals of Physics*, 191(2):363–381, 1989.