PPT^2 holds for independent states — after Collins, Yin, Zhong [arXiv:1803.00143]

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The result

Consider two independent random induced states (Model 1)

$$\rho_i := \frac{W_i}{\operatorname{Tr} W_i} = \frac{H_i H_i^*}{\operatorname{Tr}(H_i H_i^*)} \in \mathcal{M}_d(\mathbb{C}) \otimes \mathcal{M}_d(\mathbb{C}) \quad i = 1, 2,$$

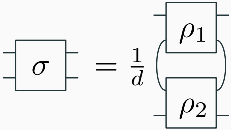
where $H_{1,2} \in \mathcal{M}_{d^2,s}(\mathbb{C})$.

Theorem (Aubrun & Aubrun-Szarek-Ye)

In the regime $4d^2 < s < Cd^3$, the states $\rho_{1,2}$ are PPT entangled.

Define the contraction

$$\sigma := \langle \Omega | \, \rho_1 \otimes \rho_2 \, | \Omega \rangle \, .$$



Theorem (Collins-Yin-Zhong)

In the regime $s^2 > cd^3 \log^2 d$, the matrix σ is approximately normalized and separable.

The proof

Lemma

If $\rho_{1,2}$ are independent induced random states of parameters $(d_1 d_2, s)$ and $\sigma = \langle \Omega_{d_1} | \, \rho_1 \otimes \rho_2 \, | \Omega_{d_1} \rangle \in \mathcal{M}_{d_2^2}(\mathbb{C})$, then, as $d_1 \to \infty$, σ is asymptotically an induced random state of parameters (d_2^2, s^2).

Proof idea: $\sigma = HH^*$, where $H \in \mathcal{M}_{d_2^2, s^2}$, with

$$H(ij, ab) = \frac{1}{\sqrt{d_1}} \sum_{x=1}^{d_1} H_1(ix, a) H_2(jx, b).$$

Use the CLT to conclude that, as $d_1 \to \infty$, H has i.i.d. standard Gaussian entries.

$\rightsquigarrow$ apply this for $d_1 = d_2 = d$ to conclude that σ is, asymptotically as $d \to \infty$, an induced random state of parameters (d^2, s^2).

Corollary

In the regime where $\rho_{1,2}$ are PPT, i.e. $s > 4d^2 > cd^{3/2} \log d$, the resulting state σ is separable.