# Applications of Sinkhorn's alternative scaling to quantum information theory

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## Problem Statement

Sinkhorn's classical result on scaling of positive matrices to bi-stochastic matrices [Sin64, SK67] has found many applications, both theoretical and practical, and has become a cornerstone method in modern computer science. The main idea is to obtain a bi-stochastic matrix (a matrix with row and column sums equal to one) by repeatedly normalizing its rows and columns. Recently, Sinkhorn's method has been generalized to the non-commutative framework by Gurvits [Gur04], the so-called *operator scaling*. The analysis of Gurvits' algorithm has only been completed recently [GGOW16], and has spawn many new research directions in non-commutative algebra, functional analysis, quantum information theory, and many other areas. A different non-commutative version of matrix scaling, called *block-matrix scaling*, was proposed in [BN17b] in relation to some properties of quantum channels. Later, block matrix scaling has found new applications in quantum group theory [BN17a]. However, rigorously proving that the block-matrix scaling algorithm terminates and the analysis of its computational complexity remain open at this time.

## Goal of the internship

The candidate will work towards a complete analysis of the block-matrix scaling algorithm, as well as research new applications of the method. A detailed comparison with operator scaling will be performed, emphasizing the similarities and differences between the two non-commutative generalizations of Sinkhorn's matrix scaling.

## Candidate's profile

The candidate should have a strong mathematical profile. Competences in linear and multilinear algebra, probability theory, and quantum (information) theory are the most important for the research project (but not strictly required). Familiarity with scientific software (`julia` or `python`) are a strong point.

## References

[BN17a] Teodor Banica and Ion Nechita. Flat matrix models for quantum permutation groups. *Advances in Applied Mathematics*, 83:24–46, 2017.

[BN17b] Tristan Benoist and Ion Nechita. On bipartite unitary matrices generating subalgebra-preserving quantum operations. *Linear Algebra and its Applications*, 521:70–103, 2017.

[GGOW16] Ankit Garg, Leonid Gurvits, Rafael Oliveira, and Avi Wigderson. A deterministic polynomial time algorithm for non-commutative rational identity testing. In *Foundations of Computer Science (FOCS), 2016 IEEE 57th Annual Symposium on*, pages 109–117. IEEE, 2016.

[Gur04] Leonid Gurvits. Classical complexity and quantum entanglement. *Journal of Computer and System Sciences*, 69(3):448–484, 2004.

[Sin64] Richard Sinkhorn. A relationship between arbitrary positive matrices and doubly stochastic matrices. *The annals of mathematical statistics*, 35(2):876–879, 1964.

[SK67] Richard Sinkhorn and Paul Knopp. Concerning nonnegative matrices and doubly stochastic matrices. *Pacific Journal of Mathematics*, 21(2):343–348, 1967.