Supervisor: Ion NECHITA
Department: LPT Toulouse

Email: nechita@irsamc.ups-tlse.fr
Webpage: https://ion.nechita.net/about

Tensor networks and
quantum max flow — min cut

1 Description

The classical maximum flow-minimum cut problem [FF56] has been an essential subject in the domain of network optimization and computer science. This project aims to extend this classic problem into the realm of quantum mechanics, particularly focusing on applications to quantum information theory [Wil17]. A comprehensive study will be carried out to establish a framework for quantum maximum flow-minimum cut problems, including novel algorithms, practical applications, and optimizations in quantum systems.

The maximum flow-minimum cut theorem is a foundational result in network flow theory. In classical systems, it has wide-ranging applications in operations research, computer science, and engineering. With the burgeoning field of quantum information theory and quantum computing, it is crucial to investigate how this classical problem generalizes to quantum systems.

Currently, the relationship between quantum network flow and the classical maximum flow-minimum cut problem is not fully understood [CFS⁺16, SSM16, Has17, GLS22]. Preliminary studies hint at a non-trivial generalization of the classical theory but lack rigorous algorithms and proofs.

Goal of the internship

Derive a mathematical framework for quantum maximum flow-minimum cut problems and quantum algorithms for solving these problems efficiently. Moreover, a secondary goal is to apply this framework to real-world problems in quantum information theory, such as quantum error correction and quantum communication.

Candidate's profile

The candidate should have a strong mathematical profile. Competences in linear and multilinear algebra, probability theory, and quantum (information) theory are the most important for the research project (but not strictly required). Familiarity with scientific software (`julia` or `python`) are a strong point.

References

[CFS⁺16] Shawn X Cui, Michael H Freedman, Or Sattath, Richard Stong, and Greg Minton. Quantum max-flow/min-cut. *Journal of Mathematical Physics*, 57(6):062206, 2016.

[FF56] Lester Randolph Ford and Delbert R Fulkerson. Maximal flow through a network. *Canadian journal of Mathematics*, 8:399–404, 1956.

[GLS22] Fulvio Gesmundo, Vladimir Lysikov, and Vincent Steffan. Quantum max-flow in the bridge graph. *arXiv preprint arXiv:2212.09794*, 2022.

[Has17] Matthew B Hastings. The asymptotics of quantum max-flow min-cut. *Communications in Mathematical Physics*, 351(1):387–418, 2017.

[SSM16] Or Sattath, Richard Stong, and Greg Minton. Quantum max-flow/min-cut. *Journal of Mathematical Physics*, 57(6), 2016.

[Wil17] Mark M Wilde. *Quantum information theory*. Cambridge University Press, 2017.